CPSC 625-600 Homework #2
SOLUTION

Total: 100 pts

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1 First-Order Logic

Important: In this section, assume that w, x, y, z are variables; A, B, C, D are constants; and $f(\cdot), g(\cdot), h(\cdot)$ are functions; and $P(\cdot), Q(\cdot), R(\cdot)$ are predicates.

1.1 Standard Forms

To do automatic theorem proving in first-order logic, you need to go through three steps to convert your initial first-order logic expression into a standard form. These are:

1. Prenex normal form,
2. Conjunctive normal form, and
3. Skolemization.

Question 1 (12 pts): Convert to prenex normal form (4 points each):

1. $\forall x \; \neg(\forall y \;\; P(x, y))$
2. $\neg\forall x\,(\neg P(x) \vee \neg(\exists y, Q(x, y)))$
3. $\neg\forall x\,((\exists y \;\; Q(x, y)) \rightarrow P(x))$

SOLUTION:

1. $\forall x \; \neg(\forall y \;\; P(x, y))$
   $\forall x \;\; (\neg\forall y \;\; P(x, y))$
   $\forall x \;\; (\exists y \neg P(x, y))$
   $\forall x \exists y \neg P(x, y)$
2. $\neg\forall x\,(\neg P(x) \vee \neg(\exists y, Q(x, y)))$
   $\exists x\,(\neg\neg P(x) \wedge \neg\neg(\exists y, Q(x, y)))$
   $\exists x\,(P(x) \wedge (\exists y, Q(x, y)))$
   $\exists x \exists y\,(P(x) \wedge Q(x, y))$

3. $\neg\forall x\,((\exists y\;\; Q(x,y)) \rightarrow P(x))$
   $\exists x \neg\,(\neg(\exists y\;\; Q(x,y)) \vee P(x))$
   $\exists x\,(\neg\neg(\exists y\;\; Q(x,y)) \wedge \neg P(x))$
   $\exists x\,((\exists y\;\; Q(x,y)) \wedge \neg P(x))$
   $\exists x \exists y\,(Q(x,y) \wedge \neg P(x))$

Question 2 (10 pts): Skolemize the expressions (2 points each):

1. $\exists x P(x)$
2. $\forall x \exists y P(x,y)$
3. $\exists x \exists y \forall z P(x,y) \wedge Q(y,z)$
4. $\forall x \exists y \forall z P(x,y) \wedge Q(y,z)$
5. $\forall x \forall y \exists z P(x,y) \wedge Q(y,z)$

SOLUTION:

1. $P(A)$
2. $P(x, f(x))$
3. $P(A,B) \wedge Q(B,z)$
4. $P(x, f(x)) \wedge Q(f(x), z)$
5. $P(x,y) \wedge Q(y, f(x,y))$

Question 3 (9 pts): Convert the following into a standard form:

$$\forall x\,[P(x) \rightarrow (\exists y Q(x,y))]$$

SOLUTION:

$$\forall x\,[\neg P(x) \vee (\exists y Q(x,y))]$$

$$\forall x \exists y\,[\neg P(x) \vee Q(x,y)]$$

$$\neg P(x) \vee Q(x, f(x))$$

1.2 Substitution and Unification

Question 1 (9 pts): Apply the following substitutions to the expressions (3 point each);

1. Apply $\{x/f(A)\}$ to $P(x,y) \vee Q(x)$.
2. Apply $\{x/A, y/f(z)\}$ to $P(x,y) \vee Q(x)$.
3. Apply $\{y/x\}$ to $P(x,y) \vee Q(x)$.

SOLUTION:

1. $P(f(A), y) \vee Q(f(A))$

2. $P(A, f(z)) \vee Q(A)$
3. $P(x, x) \vee Q(x)$

Question 2 (8 pts): For each of the following, (1) find the unifier, and (2) show the unified expression. For example, given $P(A)$ and $P(x)$, the unifier would be $\{x/A\}$, and the unified expression $P(A)$. If the pair of expressions is not unifiable, indicate so. (4 points each):

1. $P(x, f(B))$ and $P(A, f(y))$
2. $P(x, f(A))$ and $P(y, y)$
3. $P(x, f(y), y)$ and $P(A, f(g(w)), g(A))$
4. $P(A, f(y), y, A)$ and $P(x, f(g(x)), g(B), w)$

SOLUTION:

1. $\{x/A, y/B\}, P(A, f(B))$
2. $\{x/f(A), y/f(A)\}, P(f(A), f(A))$
3. $\{x/A, y/g(A), w/A\}, P(A, f(g(A)), g(A))$
4. Not unifiable.

Question 3 (8 pts): Show that $R(A)$ is a logical consequence of the following. Use **resolution**.

1. $\neg P(x) \rightarrow [Q(x, y) \vee R(y)]$
2. $\neg P(A)$
3. $\neg Q(w, z) \vee R(w)$

SOLUTION:

1. $\neg P(x) \rightarrow [Q(x, y) \vee R(y)]$
   $\neg\neg P(x) \vee [Q(x, y) \vee R(y)]$
   $P(x) \vee Q(x, y) \vee R(y)$
2. $\neg P(A)$
3. $\neg Q(w, z) \vee R(w)$
4. Negated conclusion: $\neg R(A)$
5. $P(x) \vee Q(x, A)$ (1 and 4) $\sigma = \{y/A\}$
6. $Q(A, A)$ (2 and 5) $\sigma = \{x/A\}$
7. $R(A)$ (3 and 6) $\sigma = \{w/A, z/A\}$
8. False (4 and 7)

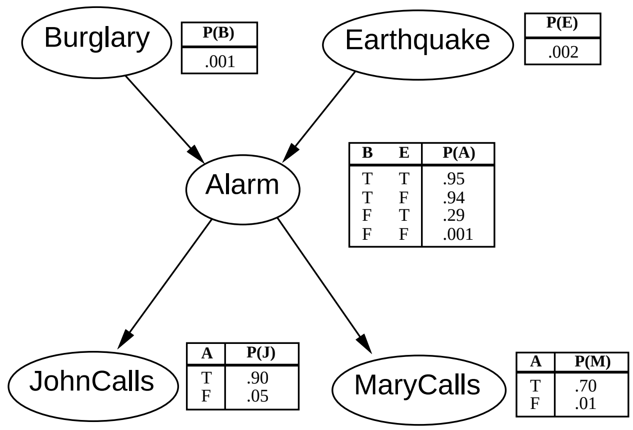

Figure 1: **Belief Network.** See problem 1.

2 Uncertainty and Probabilistic Reasoning

Question 1 (14 pts): Given the Belief network as shown in figure 1, calculate the two joint probability values and answer the question. Note that in this section $P(\cdot)$ denotes the probability of the event. (7 points each):

1. $P(\neg MaryCalls \wedge JohnCalls \wedge Alarm \wedge \neg Earthquake \wedge Burglary)$
2. $P(\neg MaryCalls \wedge JohnCalls \wedge \neg Alarm \wedge Earthquake \wedge \neg Burglary)$

SOLUTION:

1. $P(\neg MaryCalls \wedge JohnCalls \wedge Alarm \wedge \neg Earthquake \wedge Burglary)$
   $= P(\neg M|A)P(J|A)P(A|\neg E, B)P(\neg E)P(B)$
   $= (1 - 0.7) \times 0.9 \times 0.94 \times (1 - 0.002) \times 0.001$
2. $P(\neg MaryCalls \wedge JohnCalls \wedge \neg Alarm \wedge Earthquake \wedge \neg Burglary)$
   $= P(\neg M|\neg A)P(J|\neg A)P(\neg A|E, \neg B)P(E)P(\neg B)$
   $= (1 - 0.01) \times 0.05 \times (1 - 0.29) \times 0.002 \times (1 - 0.001)$

3 Learning

3.1 Decision Tree Learning

Consider the following set of examples where you are trying to make a decision whether to buy a car or not, given three decision criteria (or attributes): Resale value, Dealer location, and Type.

Example#	Resale value	Dealer location	Type	Accept Job Offer?
1	High	San Antonio	SUV	Y
2	High	Houston	Sedan	Y
3	Low	San Antonio	SUV	N
4	High	Dallas	SUV	Y
5	Medium	Dallas	SUV	N
6	Low	Dallas	Sedan	N
7	Low	Austin	Sedan	N
8	Low	San Antonio	SUV	Y
9	Low	Houston	Sedan	N
10	High	Austin	SUV	Y
11	Medium	San Antonio	Sedan	N
12	Low	Dallas	SUV	Y

Question 1 (15 pts): For each of the three attributes above, draw a decision tree rooted at that attribute with a single depth. See slide06, page 12, (a) and (b) which shows an example. (5 points each)

SOLUTION:

1. Resale
   High: (+: 1,2,4,10) (-:)
   Medium: (+:) (-: 5,11)
   Low: (+: 8,12) (-: 3,6,7,9)

2. Location
   San Antonio: (+: 1, 8) (-: 3, 11)
   Houston: (+: 2) (-: 9)
   Dallas: (+: 4, 12) (-: 5, 6)
   Austin: (+: 10) (-: 7)

3. Type
   SUV: (+: 1, 4, 8, 10, 12) (-: 3, 5)
   Sedan: (+: 2) (-: 6, 7, 9, 11)

Question 2 (15 pts): Calculate the information gain for each of the three attributes and explain which attribute should be picked to be tested first. (5 points each)

SOLUTION:

1. Define $Ent(x, y) = -x \log_2 x - y \log_2 y$.
   $Ent(6/12, 6/12) - (4/12Ent(4/4, 0/4) + 2/12Ent(0/2, 2/2) + 6/12Ent(2/6, 4/6)$
   $= 1 - 0.46915 = 0.54085$

2. $Ent(6/12, 6/12) - (4/12Ent(2/4, 2/4) + 2/12Ent(1/2, 1/2) + 4/12Ent(2/4, 2/4) + Ent(1/2, 1/2)$
   $= 1 - 1 = 0$

3. $Ent(6/12, 6/12) - (7/12Ent(5/7, 2/7) + 5/12Ent(1/5, 4/5)$
   $= 1 - 0.80429 = 0.19571$