CPSC 625-600 Homework #2
Due 11/05/ (Monday) 3pm

Handwritten or printed hardcopy must be submitted to the TA
Total: 100 pts

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1 First-Order Logic

Important: In this section, assume that w, x, y, z are variables; A, B, C, D are constants; and $f(\cdot), g(\cdot), h(\cdot)$ are functions; and $P(\cdot), Q(\cdot), R(\cdot)$ are predicates.

1.1 Standard Forms

To do automatic theorem proving in first-order logic, you need to go through three steps to convert your initial first-order logic expression into a standard form. These are:

1. Prenex normal form,
2. Conjunctive normal form, and
3. Skolemization.

Question 1 (12 pts): Convert to prenex normal form (4 points each):

1. $\forall x \ \neg (\forall y \ \ P(x, y))$
2. $\neg \forall x \, (\neg P(x) \vee \neg (\exists y, Q(x, y)))$
3. $\neg \forall x \, ((\exists y \ \ Q(x, y)) \rightarrow P(x))$

Question 2 (10 pts): Skolemize the expressions (2 points each):

1. $\exists x P(x)$
2. $\forall x \exists y P(x, y)$
3. $\exists x \exists y \forall z P(x, y) \wedge Q(y, z)$
4. $\forall x \exists y \forall z P(x, y) \wedge Q(y, z)$
5. $\forall x \forall y \exists z P(x, y) \wedge Q(y, z)$

Question 3 (9 pts): Convert the following into a standard form:

$$\forall x \, [P(x) \rightarrow (\exists y Q(x, y))]$$

1.2 Substitution and Unification

Question 1 (9 pts): Apply the following substitutions to the expressions (3 point each);

1. Apply $\{x/f(A)\}$ to $P(x, y) \vee Q(x)$.
2. Apply $\{x/A, y/f(z)\}$ to $P(x, y) \vee Q(x)$.
3. Apply $\{y/x)\}$ to $P(x, y) \vee Q(x)$.

Question 2 (8 pts): For each of the following, (1) find the unifier, and (2) show the unified expression. For example, given $P(A)$ and $P(x)$, the unifier would be $\{x/A\}$, and the unified expression $P(A)$. If the pair of expressions is not unifiable, indicate so. (4 points each):

1. $P(x, f(B))$ and $P(A, f(y))$
2. $P(x, f(A))$ and $P(y, y)$
3. $P(x, f(y), y)$ and $P(A, f(g(w)), g(A))$
4. $P(A, f(y), y, A)$ and $P(x, f(g(x)), g(B), w)$

Question 3 (8 pts): Show that $R(A)$ is a logical consequence of the following. Use **resolution**.

1. $\neg P(x) \rightarrow [Q(x, y) \vee R(y)]$
2. $\neg P(A)$
3. $\neg Q(w, z) \vee R(w)$

2 Uncertainty and Probabilistic Reasoning

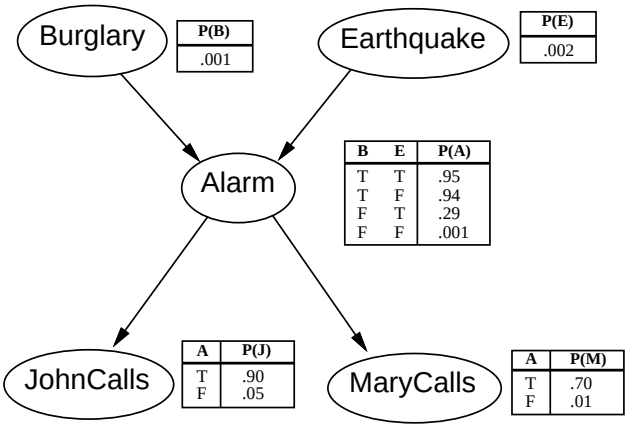

Figure 1: **Belief Network.** See problem 1.

Question 1 (14 pts): Given the Belief network as shown in figure 1, calculate the two joint probability values and answer the question. Note that in this section $P(\cdot)$ denotes the probability of the event. (7 points each):

1. $P(\neg MaryCalls \wedge JohnCalls \wedge Alarm \wedge \neg Earthquake \wedge Burglary)$
2. $P(\neg MaryCalls \wedge JohnCalls \wedge Alarm \wedge Earthquake \wedge \neg Burglary)$

3 Learning

3.1 Decision Tree Learning

Consider the following set of examples where you are trying to make a decision whether to buy a car or not, given three decision criteria (or attributes): Resale value, Dealer location, and Type.

Example#	Resale value	Dealer location	Type	Accept Job Offer?
1	High	San Antonio	SUV	Y
2	High	Houston	Sedan	Y
3	Low	San Antonio	SUV	N
4	High	Dallas	SUV	Y
5	Medium	Dallas	SUV	N
6	Low	Dallas	Sedan	N
7	Low	Austin	Sedan	N
8	Low	San Antonio	SUV	Y
9	Low	Houston	Sedan	N
10	High	Austin	SUV	Y
11	Medium	San Antonio	Sedan	N
12	Low	Dallas	SUV	Y

Question 1 (15 pts): For each of the three attributes above, draw a decision tree rooted at that attribute with a single depth. See slide06, page 12, (a) and (b) which shows an example. (5 points each)

Question 2 (15 pts): Calculate the information gain for each of the three attributes and explain which attribute should be picked to be tested first. (5 points each)