Complex Dynamics Is Abolished in Delayed Recurrent Systems with Distributed Feedback Times

by Thiel et al. (2003)

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Introduction

- Feedback systems with a single delay time: known to exhibit various dynamical behaviors including complex oscillations and chaos.
- With broad distribution of delays, yields a larger set of parameter values that results in fixed point behavior or simple oscillatory behavior.

Background

- In time-lagged recurrent feedback systems, feedback gain and delay may serve as a bifurcation parameter whose increase yields a sequence of bifurcations leading from fixed point behavior to periodic orbits, and finally chaos.
- In most studies, recurrent signals are assumed to come from a singular instant in the past.
- However, in biological systems, there may be a wider range of delay in the feedback.

Basic Concept: Bifurcation

- Logistic map: \( x = a \times x \times (1 - x) \).
- With random initial values \( x_0 \), calculate sequence of \( x \)'s, and find the steady-state.
- Plot the steady states for different parameter values: Bifurcation diagram.
**Approach**

- Build up from existing models (with singular delay) showing complex dynamic.
  - Inhibitory feedback in hippocampus.
  - Mackey-Glass equation (regulation process of white blood cells).
  - Logistic growth of an ecological population under resource limits.
- Introduce distributed delay and observe resulting change in behavior.

**Neural Feedback in the Hippocampus**

- Mossy fibers (exc) → CA3 pyramidal cell (exc) → interneuronal basket cells (inh) → CA3 pyramidal cell
- Delay in the feedback inhibitory loop can vary.
- Amount of feedback may also affect dynamic behavior: penicillin can modulate this (GABA antagonist).
- Model:
  \[
  \frac{dv(t)}{dt} = -\Gamma v(t) + \Gamma e - \beta \frac{F_\xi(v(t))}{1 + F_\xi(v(t))^n},
  \]
  \[
  F_\xi(v(t)) = f_0 \int_0^\infty [v(t - \tau) - \theta] + \xi(\tau) d\tau.
  \]

- Some assumptions:
  \[
  \int_0^\infty \xi(\tau) d\tau = 1
  \]
- Simplest form:
  \[
  \xi(\tau) = \begin{cases} 
  1/2\sigma & \text{if } \tau_m - \sigma \leq \tau \leq \tau_m + \sigma \\
  0 & \text{otherwise}
  \end{cases}
  \]

**Results: Hippocampus Model with Singular Delay**

- Singular delay.
- Bifurcation parameter \( \beta \) increased from top to bottom.
- Complex dynamic results.
Results: Hippocampus Model with Distributed Delay

- Various delay distributions.
- Dynamic becomes simpler.

Mackey-Glass System: White Blood Cell Regulation

- Production of neutrophil granulocytes (a type of white blood cell).
- Production depends on present amount, but new production matures with a delay.
- Altered feedback gain or delay causes period-doubling bifurcations leading to chaos: Suspected cause of chronic granulocytic leukemia.

Mackey-Glass System

- Normalized concentration of cells:
  \[
  \frac{dv(t)}{dt} = -\gamma v(t) + \beta \frac{V_\xi(v(t))}{1 + V_\xi(v(t))^n},
  \]
  \[
  V_\xi(v(t)) = \int_0^\infty v(t - \tau)\xi(\tau) d\tau,
  \]
  \(\gamma\): cell loss rate; \(\beta\): gain in regulation; Same \(\xi\) as before.
Results: Mackay-Glass Model with Distributed Delay

- More distributed delay gives simpler dynamic.
- Bar indicates the integration interval.

Population Dynamics

- Typical model is the logistic equation (introduced earlier).
- Individual maturation time may differ, causing a spread in the delay distribution.

Population Density

- Delay-differential equation for population density $N$:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{1}{K}N\xi(N(t))\right),$$

$$N\xi(N(t)) = \int_0^\infty N(t - \tau)\xi(\tau)d\tau,$$

$$\xi(\tau) = \begin{cases} 
0 & \text{if } 0 \leq \tau \leq \tau_{\min} \\
(\tau - \tau_{\min}) \exp\left(-\frac{(\tau - \tau_{\min})}{\theta}\right)/\theta^2 & \text{if } \tau > \tau_{\min}
\end{cases}$$

Mean delay: $\tau_m = \tau_{\min} + 2\theta$; Variance in delay: $\sigma^2 = 2\theta^2$.

Population Density for Different Delay Distributions

- High-amplitude oscillation is not good due to the risk of extinction during low-population periods.
- Distributed delay causes population to stabilize into a stable equilibrium.
Summary and Discussions

- Increasing the spread of delay distribution has a profound effect of dynamics in biological systems.
- Why does the dynamic become simpler in this case?: smoothing, reduced variance.
- Integration interval is shorter than period of oscillation, so there’s no over-smoothing.
- The observed effects are robust.

References