Relationship Between Visual Cortical Response Powerlaw and Perceptual Threshold

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What Is Common in These Images?

• In color, natural image, from the Kodak data set, ...
  – What about the brightness intensity histogram?

What Is Similar Then?

• They are very different!
The Visual Cortical Response

- Retina: center-surround filter
- LGN (thalamus): center-surround filter
- Visual cortex: oriented Gabor filter

Simulating Visual Cortical Response: Convolution with Oriented Gabor Filters

- Oriented Gabor filters simulate visual cortical receptive fields.

**Visual Cortical Response (Simulated)**

- This is (sort of) how the visual cortex responds to these images (Gabor filtering [next slide]).
  - Oriented edges are most prominently detected.
  - Would the response histogram vary as much as the brightness intensity histogram?

**Visual Cortical Response Histogram**

- The response (called orientation energy $E$) distributions are similar across the board!
- **Power law** property is observed (this is already a well-known result; Field 1987): $f(x) = 1/x^a$ ($a > 0$).
Yet Another Power Law!

Power law seems to be ubiquitous in nature and in human-made artifacts:
- 957,000 documents returned by Google Scholar!
- Power law phenomena range from www topology, financial market fluctuation, to word frequency and much more (see e.g., Clauset et al. 2009).

However, it is not often asked:
- What use is it?
- What fundamental mechanisms underlie such phenomena?

Part I:
Power Law + Gaussian Baseline = Human Perceptual Threshold

Insight: Comparing the power law distribution with a normal distribution with the same variance can be useful.

- Assumption: normal distribution can be a suitable baseline.

The point $L_2$ where $h(E)$ becomes greater than $g(E)$ may be important, i.e., orientation energy is suspiciously high.
Can there be a relationship between the threshold of $E$ above which humans see it as salient and the point $L2$?

- **Experiment**: Human participant selected threshold of $E$ so that (1) contours are preserved as much as possible and (2) noise reduced as much as possible.

Further Discoveries: $L2$ and Response Std. Dev.

Further, the raw standard deviation $\sigma$ of the response distribution is linearly related to $L2$.

- Question: Is there an analytical solution to
  \[ a \frac{1}{x^b} = c \times \exp\left(-\frac{x^2}{d}\right) \]
  where the constants $a$, $b$, $c$, and $d$ depend on $\sigma$? (more on this later)

Using $\sigma$ to Estimate Optimal $E$ Threshold

Relating $\sigma$ back to the human-chosen $E$ threshold gives again a linear relation:

\[ T_\sigma = 1.37\sigma - 2176.59. \]

Thus, instead of calculating the histogram, etc., we can simply calculate the raw standard deviation $\sigma$ to estimate the appropriate $E$ threshold.
Three Quantities

An unexpected correlation found among:

- Human-selected threshold.
- \( L_2 \), point of intersection of response power law and Gaussian baseline.
- \( \sigma \), standard deviation of response power law.

Application: Thresholding Cortical Response

- Original, human-selected, 85-percentile, and \( T_\sigma \).

Thresholding Cortical Response

- \( T_\sigma \) as a threshold gives good results, comparable to humans’ preference.

Thresholding: Limitations of Fixed Percentile

- Original, human-selected, 85-percentile, and \( T_\sigma \).
Thresholding $E$: Limitations of Global Thresholding

- Original, human-selected, 85-percentile, $T_\sigma$, and $T_\sigma$ local.
- Estimating $T_\sigma$ at a local scale solves the problem.

Part I: Summary

- Visual cortical response exhibits a power law.
- Comparing the power law to a baseline normal distribution results in a quantity ($L^2$) that is linearly correlated with human perceptual threshold.
- $L^2$ is in turn linearly correlated with the standard deviation of the power law.
- Straight-forward application possible (thresholding, salient edge detection):
  - Simple calculation of response variance is enough!

Part II: Why the Gaussian Baseline?

- The results are promising, but why?
- Why is the normal (Gaussian) distribution a reasonable choice as a baseline?
  - Central limit theorem?
  - People commonly use it?
Power Law, Gaussian Dist., vs. Suspicious Coincidence

- What is the relationship between salience defined as super-Gaussian and the conventional definition of suspiciousness (Barlow 1994, 1989)?

\[ P(A, B) > P(A)P(B), \]

where \( A \) and \( B \) are pixels in an image.

White-Noise Analysis

- In white-noise images, each pixel is independent, so, given any pixel pair \( (A, B) \):

\[ P(A, B) = P(A)P(B). \]

- Would we get a power law response?
  - If the Gaussian baseline assumption was correct, since there is no salient edge, the response distribution should be Gaussian.

Visual Response to White Noise Images

- The orientation energy distribution is very close to a Gaussian, especially near the high \( E \) values.
- Thus, the \( T_\sigma \) thresholding will not produce a meaningful threshold.

Use of White Noise Response as a Baseline

- Can we use the white-noise response as a baseline for thresholding \( E \)?: Yes!
- Generate white noise response, and scale it by \( \sigma_h/\sigma_r \) where \( \sigma_h \) and \( \sigma_r \) are the STD in the natural image response and the white noise response.
- Recalculate the response distribution (if necessary).
**New Baseline for Salience vs. Humans**

New $L_2$ vs. Human Chosen Threshold ($r = 0.98$)*

- Strong linearity is found between the new $L_2$ and the human selected threshold.
  - * This is much tighter than the Gaussian baseline ($r = 0.91$)!

**Part II: Summary**

- Gaussian baseline corresponds to response distribution to white noise images.
- In white noise images, each pixel is independent from the others.
- This relates to the idea of suspicious coincidence by Barlow (1994)
- Threshold derived using the white-noise response distribution is even more accurate than earlier results.

**New Baseline for Salience vs. $\sigma$**

New $L_2$ vs. $\sigma$ ($r = 0.91$)

- The same linearity between $L_2$ and the $\sigma$ is maintained.

**Part III: Deeper Questions**
Neural Implementation

- The local (or even global) threshold calculation can be easily implemented in a neural circuit:

$$\sigma^2 = \sum_{i,j} w_{ij} g(V_{ij}),$$

where $w_{ij}$ are connection weights serving as normalization constants, $g(x) = x^2$, and $V_{ij}$ is the V1 response at location $i, j$.

- The resulting value can be passed through another activation function $f(x) = \sqrt{x}$.

$$f(\sigma^2) = \sqrt{\sigma^2} = \sigma$$

- These are all plausible functions that can be implemented in a biological neural network.

Mathematical/Statistical Implications

Is there an analytical solution to $a \frac{1}{x^b} = c \times \exp(-\frac{x^2}{d})$?

- This leads to another obscure yet surprisingly ubiquitous function called the Lambert W function $W(x)$ which is defined as the inverse of the following function:

$$x = W \exp(W)$$

- The Lambert W function is popping up everywhere: delay differential equations (with applications in population dynamics, economics, control theory), projectile trajectory calculation, voltage/current/resistance in a diode, etc. (see Hayes 2005 for a review)—A déjà vu?

- Speculation: Power law, Gaussian, and Lambert W function are deeply related.

Power Law, Gaussian, and Lambert W function

- Basically, $x = \pm ip \sqrt{W(-q)}$

- How I found out: Wolfram Alpha (Mathematica, prior to that).

Wrap Up
Related Work

- Malik et al. (Malik et al. 1999) used peak values of orientation energy to define boundaries of regions of coherent brightness and texture.
- The non-Gaussian nature of orientation energy (or wavelet response) histograms has also been recognized and utilized, especially in denoising and compression (Simoncelli and Adelson 1996).
- Other kinds of histograms, e.g., spectral histogram by Liu and Wang (2002), or spatial frequency distributions (Field 1987), may be amenable to a similar analysis.

Conclusions

- Visual cortical response shows a power law.
- Power law compared to Gaussian baseline gives accurate predictor for human perceptual threshold.
- Standard deviation of the response is a simple yet powerful approximation.
- Gaussian baseline found to be related to suspicious coincidence.
- Power law, Gaussian baseline, and Lambert W function intricately interrelated.
- **Lesson:** Power law is there for a reason, and it can greatly simplify things downstream.

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References


