1 Uninformed Search

Consider the search tree in Fig. 1. Assume that the exploration of the children of a particular node proceeds from the left to the right for all search methods in this section.

**Question 1 (4 pts):** Give an example of when breadth-first search (BFS) and depth-first search (DFS) have the same time complexity. Pick one goal node number from the tree above where such a case happens.

**Question 2 (4 pts):** Give an example when depth first search is suboptimal. Pick two node numbers as goal nodes as an example.

**Question 3 (4 pts):** What limitation in BFS does iterative deepening search overcome?

**Question 4 (4 pts):** Why is the space complexity of BFS $O(b^d+1)$, not $O(b^d)$, where $b$ is the branching factor and $d$ is the goal depth?

**Question 5 (4 pts):** Can depth limited search become incomplete in the case of the finite search tree above? If so, give an example. If not, explain why not.

2 Informed Search

**Question 6 (10 pts):** Manually conduct greedy best-first search on the graph below (Fig. 2), with initial node $a$ and goal node $m$. Actual cost from node to node are shown as edge labels. The heuristic function value for each node is shown in a separate table to the right. Show:

1. Node list content at each step
2. Node visit order
3. Solution path
4. Cost of the final solution.

**Note:** For A*, you may need to track which path you followed to reach node \( n \) and calculate the \( f(n) \) value accordingly. It helps to write the node \( h \) as \( a_d h \) to indicate the path in subscript (\( a \rightarrow d \rightarrow h \)).

**Question 7 (10 pts):** (1) Repeat the problem right above with A* search. (2) In addition, show the \( f(n) \) value for all nodes expanded (you need this to sort them in the node list). (3) Which one gives a shorter solution: Greedy best-first or A*? **Note:** Note that the same node can appear in the node list with a different \( f(n) \) value, depending on the path taken.

**Question 8 (12 pts):** Explain why A* is optimal, in the general case. Explain in terms of an arbitrary node \( n \) on the path to an optimal goal \( G_1 \), and a separate suboptimal goal \( G_2 \).
3 Game Playing

3.1 Minmax Search

Question 9 (4 pts): Using the following figure 3, use minmax search to assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. Assume you explore the successors from left to right.

![Game Tree](image)

Figure 3: Game Tree. Solve using minmax search.

3.2 $\alpha - \beta$ pruning

Question 10 (8 pts): Using the following figure 4, use $\alpha - \beta$ pruning to (1) assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. (2) For each node, indicate the final $\alpha$ and $\beta$ values. (Note that initial values at the root are $\alpha = -\infty, \beta = \infty$.) (3) For each cut that happens, draw a line to cross out that subtree.

![Game Tree](image)

Figure 4: Game Tree. Solve using $\alpha - \beta$ pruning. This tree is the same as figure 3.

Question 11 (6 pts): In Minmax search, we used a depth-first exploration through the use of recursion. We know that Minmax gives an optimal solution, however, we also know that depth-first search is suboptimal. Explain why Minmax gives an optimal solution even when it is using a depth-first exploration.
4 Logic and Theorem Proving

4.1 Normal forms

In all of the problems in this section, show each step of the derivation and indicate which axioms (or other rules) you used: For example, distributive law, by definition, etc.

**Question 12 (2 pts):** Convert $\neg(S \rightarrow P) \lor (\neg((Q \land \neg R) \rightarrow \neg S))$ into conjunctive normal form.

**Question 13 (2 pts):** Convert $\neg(R \land ((P \rightarrow \neg Q) \rightarrow \neg S)) \rightarrow T$ into disjunctive normal form.

**Question 14 (8 pts):** Convert the following into prenex normal form, and then into conjunctive normal form, and then skolemize.

1. $\neg\forall x (P(x) \rightarrow \neg(\exists y, Q(x, y)))$
2. $\forall x \neg ((\neg\forall y \neg Q(x, y)) \rightarrow P(x))$

4.2 Theorem proving

**Question 15 (18 pts):** Show that $\exists x (Q(x) \land R(x))$ is a logical consequence of the following, using resolution.

1. $P(a)$
2. $Q(a)$
3. $Q(y) \lor S(a, y)$
4. $\neg Q(x) \lor \neg V(x)$
5. $\neg P(x) \lor V(x) \lor S(x, f(x))$
6. $\neg P(x) \lor V(x) \lor R(f(x))$

Hint: first, transform the problem into a set of clauses, and then follow the resolution steps. Don’t forget the negate to conclusion.