Boltzmann Learning

- Stochastic learning algorithm rooted in statistical mechanics.
- Recurrent network, binary neurons (on: '+1', off: '-1').
- Energy function E:

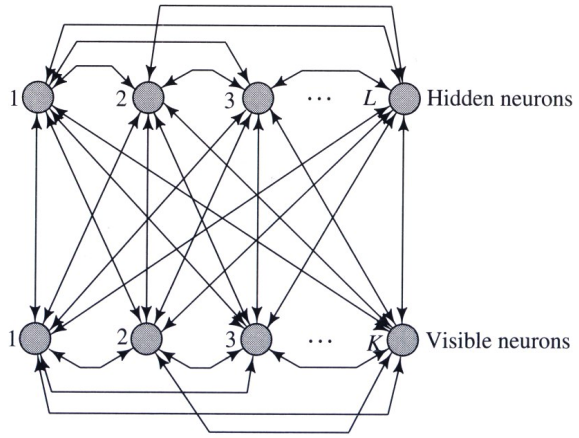
$$E = -\frac{1}{2} \sum_{j} \sum_{k, k \neq j} w_{kj} x_k x_j$$

- Activation:
  - Choose a random neuron k.
  - Flip state with a probability (given temperature T)

$$P(x_k \rightarrow -x_k) = \frac{1}{1 + \exp(-\Delta E_k / T)}$$

where ΔE_k is the change in E due to the flip.

Boltzmann Machine



- Two types of neurons
  - **Visible** neurons: can be affected by the environment
  - **Hidden** neurons: isolated
- Two modes of operation
  - **Clamped**: visible neuron states are fixed by environmental input and held constant.
  - **Free-running**: all neurons are allowed to update their activity freely.

Boltzmann Machine: Learning and Operation

- Learning:
  - Correlation of activity during clamped condition ρ_{kj}^{+}
  - Correlation of activity during free-running condition ρ_{kj}^{-}
  - Weight update: $\Delta w_{kj} = \eta(\rho_{kj}^{+} - \rho_{kj}^{-}), j \neq k$.
- Train weights w_{kj} with various clamping input patterns.
- After training is completed, present new clamping input pattern that is a partial input of one of the known vectors.
- Let it run clamped on the new input (subset of visible neurons), and eventually it will complete the pattern (**pattern completion**).

$$\text{Correl}(x, y) = \frac{\text{Cov}(x, y)}{\sigma_x \sigma_y}$$